Erratum: Patterning Electro-osmotic Flow with Patterned Surface Charge [Phys. Rev. Lett. 84, 3314 (2000)]

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DOI: 10.1103/PhysRevLett.86.6050 PACS numbers: 47.54.+r, 47.65.+a, 61.20.Lc, 82.45.–h, 99.10.+g

There were errors in Eqs. (2)–(5) in our original Letter. The correct forms of these equations are presented in this erratum. As a consequence of the errors in Eqs. (3), (4), and (5), the estimate of the value μ_w given in the second paragraph of page 3317 of the original article was incorrect. With the correct form of these equations, $\mu_w = -0.37$ (μm/s)/(V/cm) not -0.55 (μm/s)/(V/cm) gives the best fit of the model flow in Eq. (3) to the experimental measurements. All other results and conclusions in the original article were correct.

Given the definition of the electro-osmotic mobility of charged walls, $\mu = \sigma/\eta\kappa$ in the first paragraph of page 3315, the slip boundary condition defined in the same paragraph should contain a minus sign: $v(x_0, y_0, z) = -\mu(x_0, y_0, z)E$.

Equation (2) predicts the velocity of beads with electro-osmotic (EO) mobility μ_b in the EO flow with $\mathbf{q} \perp \mathbf{E}$. Taking into account the correct definition of the EO velocity, and the dimensions of the channel given in Fig. 1, Eq. (2) should be

$$\begin{aligned}\mathbf{v}(x,y) = v_z\mathbf{e}_z &= [\mu_b E - (1/2)(\mu_+ + \mu_-)E]\mathbf{e}_z - (1/2)(\mu_+ - \mu_-)E \\ &\times \sum_{n\,\mathrm{odd}}^{\infty} (4/n\pi)\{\mathrm{csch}(2n\pi a/b)[\sinh(2n\pi y/b) - \sinh\mathbf{(}2n\pi(y-a)/b\mathbf{)}]\sin(2n\pi x/b) \\ &\qquad + \mathrm{csch}(n\pi b/2a)\sinh(n\pi x/a)\sin(n\pi y/a)\}\mathbf{e}_z\,. \end{aligned} \tag{2}$$

The errors in Eq. (2) as presented in the article were not present in the calculations used to fit the model to the experimental measurements, so there were no changes in the estimates of the experimental parameters that were presented in the first paragraph on page 3316.

Equation (3) that predicts the stream function for the observed motion of the fluorescent beads in the flow with $\mathbf{q} \parallel \mathbf{E}$ should be

$$\begin{aligned}\varphi(x,y,z) &= Ef_c(x,y) + E\sum_{m\,\mathrm{odd}}^{\infty}(-1)^{(m-1)/2}\cos(m\pi x/b) \\ &\times \Bigg(\{[(\mu_+\lambda_+ + \mu_-\lambda_-)/(\lambda_+ + \lambda_-)]b(2/m\pi)^2\mathrm{csch}(m\pi a/b)\cosh[m\pi(a-y)/b]\} + \tfrac{1}{2}(\mu_+ - \mu_-)(4/m\pi) \\ &\quad \times \sum_{n=1}^{\infty}\{(4/n\pi)\sin(q_n\lambda_+/2)\cos(q_n z)[c_1\sinh(q_n y) + c_2 y e^{-q_n y} + c_3 y e^{q_n y}]\}\Bigg).\end{aligned} \tag{3}$$

The function f_c that is defined below Eq. (3) on page 3315 represents, in the form of a stream function, the flow due to the unpatterned walls and the electrophoretic motion of the tracer beads. The function f_c should be $f_c(x,y) = -[y\mu_{\mathrm{beads}} + \mu_{\mathrm{walls}}f_w]$.

In Eq. (4), $f_w(x,y)$ should be the negative of the integral with respect to y of the axial flow generated by the unpatterned walls. In the original article, f_w was the flow itself, and yf_w was used to represent the stream function of this flow. The error due to this mistake was small because the flow due to the unpatterned walls varies slowly over most of the cross section of the channel. Equation (4) should be

$$\begin{aligned}f_w(x,y) = \sum_{n\,\mathrm{odd}}^{\infty}(2/n\pi)^2\mathbf{(}&a\,\mathrm{csch}(n\pi b/a)\{\sinh[n\pi(x + b/2)/a] - \sinh[n\pi(x - b/2)/a]\} \\ &\times \cos(n\pi y/a) + b(-1)^{(n+1)/2}\mathrm{csch}(n\pi a/b)\cosh(n\pi y/b)\cos(n\pi x/b)\mathbf{)}\,.\end{aligned} \tag{4}$$

Equation (5) should be

$$\begin{aligned} c_1 &= 2(q_n a)^2/q_n[1 + 2(q_n a)^2 - \cosh(2q_n a)], \\ c_2 &= (-e^{2q_n a} + 2q_n a + 1)/2[1 + 2(q_n a)^2 - \cosh(2q_n a)], \\ c_3 &= \frac{[\sinh(q_n a) - \cosh(q_n a)][q_n a\cosh(q_n a) + (q_n a - 1)\sinh(q_n a)]}{1 + 2(q_n a)^2 - \cosh(2q_n a)} \end{aligned} \tag{5}$$

With these changes in Eqs. (3)–(5), the best fit between the model and the measured motion of the beads is achieved for $\mu_w = -0.37$ (μm/s)/(V/cm).

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